Scattering of Internal Gravity Waves at Finite Topography

Peter Muller
University of Hawaii
Department of Oceanography
1000 Pope Road, MSB 429
Honolulu, HI 96822
phone: (808)956-8081 fax: (808)956-9164 email: pmuller@soest.hawaii.edu

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LONG-TERM GOAL

The long term goal of the research project is the construction of a numerical model that

- predicts the internal wave field and internal wave induced transports regionally and globally, and
- can be used in conjunction with circulation, turbulence, acoustic and other models for research applications.

OBJECTIVES

The internal wave model will be based on the integration of the radiation balance equation. The current project explores some of the basic questions that arise in the design of such a model:

- Can the representation of the internal wave field and its effect on other motions be improved by the fact that internal waves represent that part of the motion that carries no potential vorticity?
- What errors are caused when one uses internal wave dispersion relations that are only locally valid and that neglect the meridional component of the earth's rotation?
- Which parts of the ocean are "illuminated" by wave group trajectories that emanate from points at the boundaries of the ocean? This question addresses whether or not wind-generated near-inertial internal waves and topographically generated baroclinic tides can fill out ocean basins and be the sole sources of internal wave energy in the ocean.

APPROACH

The radiation balance equation describes changes of the action density spectrum of the internal wave field along wave group trajectories caused by generation, transfer, and dissipation processes. The predicted quantity will be the action density spectrum as a function of wavenumber, position, and time. Overall, the project will emulate the WAM project for surface gravity waves in its approach and methodology.

The radiation balance equation makes three basic approximations:

(1) the random phase approximation

(2) the WKB or geometric optics approximation

(3) the weak interaction approximation.

The random phase approximation assumes that the various dynamical processes affecting the internal wave field will distort the wave phases in an irregular way such that it is neither possible nor desirable to predict the wave phases. Instead the wave field is described by its energy or action density spectrum.

The WKB approximation assumes that the wavelengths of the waves are small compared to the scales of the environment. The action density spectrum as a function of wavenumber then varies slowly with position and time on the scales of the environment.

The weak interaction approximation assumes that internal waves are basically a linear phenomenon. The waves propagate along their wave group trajectories, being only slowly modified by dynamical processes. The dynamical evolution of the action density spectrum is then governed by the radiation balance equation.

WORK COMPLETED

We are currently investigating three problems:

(1) Internal gravity waves as the zero potential vorticity mode. Internal gravity waves do not carry potential vorticity. There exists a general decomposition of the velocity into a part that carries potential vorticity (this is the vortical mode) and a part that does not carry potential vorticity (this is the gravity mode). This decomposition is valid for arbitrary flows in a nonhomentropic ideal fluid, which includes flows in a stratified Boussinesq fluid. It does not require any simplifying assumptions. The dynamical equations that describe the time evolution of the vortical and gravity mode are most easily obtained in an Eulerian form of Hamilton's principle. Unfortunately, these equations contain "non-physical" variables that obscure the consequences of the decomposition for the problem of internal wave propagation and the parameterization of internal wave induced fluxes.

(2) Effects of the meridional component of earth's rotation. Here we quantify the absolute and relative error that the traditional approximation, i.e., the neglect of the meridional component of the earth's rotation has on the frequency, group velocity, and rate of refraction of internal gravity waves. The absolute error is at most of the order of the earth's rotation rate. Traditional scaling arguments suggest that the absolute error becomes smaller for near-inertial internal waves implying the hydrostatic approximation. For high frequency waves with frequencies close to the buoyancy frequency the relative error becomes small. However, there are medium range frequencies for which the effect is important and includes not only significant changes in the frequency but also changes in propagation direction and speed.

(3) The illumination problem. Internal waves propagate along their wave group trajectories in six-dimensional position- wavenumber space. The trajectories are determined by six coupled ordinary differential equations. Changes in position are given by the group velocity, changes in the wavenumber by the rate of refraction. At boundaries reflection laws apply. We are shining

Garrett and Munk spectra on various types of topography, mostly slope-shelf configurations. We determine where internal waves are traveling and how much energy flux they carry with them. From this upper bounds for internal wave induced mixing can be obtained. For simple configurations, we compare the results of the WKB trajectories with the more exact results from diffractive calculations.

RESULTS

All three investigations are ongoing and no final results have been obtained as yet.

IMPACT/APPLICATION

The development of a predictive dynamical model of the global or regional internal wave fields will have many benefits and applications.

Internal wave research will benefit from such a model since

- it will provide understanding of the internal wave field as a balance of generation, transfer and dissipation processes,
- it will focus research (it is expected that the proposed model will do for internal wave dynamics what the GM model did for internal wave kinematics), and
- it will predict changes of the internal wave field in response to changes in the forcing and environmental fields.

The dynamical internal wave model can also be run in conjunction with circulation models, turbulence models, chemical tracer models, and biological population models where it would predict the internal wave induced transports, dispersion and mixing. In conjunction with acoustic transmission models the model would predict the internal wave induced "noise."

TRANSITIONS

None

RELATED PROJECTS

None

REFERENCES

PUBLICATIONS

Müller, P. and X. Liu, 2000: Scattering of internal waves at finite topography in two dimensions. Part II: Spectral calculations and boundary mixing. *J. Phys. Oceanogr.* **30**, 550-563.

Müller, P. and X. Liu, 2000: Scattering of internal waves at finite topography in two dimensions. Part I: Theory and case studies. *J. Phys. Oceanogr.* **30**, 532-549.

Wang, D. and P. Müller, 1999: Internal wave generation in the equatorial undercurrent, In: "Dynamics of Internal Gravity Waves, II." Proceedings, 'Aha Huliko'a Hawaiian Winter Workshop, School of Ocean and Earth Science and Technology, Special publication, 181-187.

Müller, P. 1999: On redistributed energy fluxes in topographic scattering problems. In: "Dynamics of Internal Gravity Waves, II" Proceedings, 'Aha Huliko'a Hawaiian Winter Workshop, School of Ocean and Earth Science and Technology, Special publication, 155-162.